PERGAMON

International Journal of Heat and Mass Transfer 43 (2000) 4361–4364

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

Simultaneous estimation of the profiles of the temperature and the scattering albedo in an absorbing, emitting, and isotropically scattering medium by inverse analysis

Huai-Chun Zhou*, Ping Yuan, Feng Sheng, Chu-Guang Zheng

National Laboratory of Coal Combustion, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China

Received 28 October 1999; received in revised form 4 February 2000

Radiative heat transfer is important in high temperature devices such as combustion chambers and furnaces. Inverse radiation problems are concerned with the determination of the radiative properties, such as the scattering albedo, the phase function, or the optical thickness, and the source profile of the medium [1–3]. One-dimensional radiative heat transfer has been extensively reviewed in literature [4]. The temperature profile in an absorbing, emitting, and isotropically scattering medium has been identified by inverse analysis [5]. We will use the conjugate gradient method of minimization [5] and the two-color method [6] to solve an inverse radiation problem for estimating simultaneously the temperature profile and the scattering albedo profile in an absorbing, emitting, and isotropically scattering medium.

1. Method for approach

We consider an absorbing, emitting, isotropically scattering, gray plane–parallel medium of optical thickness $\tau_0 = 1.0$, and azimuthally symmetric radiation. The bounding surfaces at $\tau = 0$, $\tau = \tau_0$ are transparent and there is no externally incident radiation. The equation of radiative transfer has been treated in [5] as

$$\mu \frac{\partial I(\tau, \mu)}{\partial \tau} + I(\tau, \mu) = S(\tau) + \frac{\bar{\omega}}{2} \int_{-1}^{1} I(\tau, \mu')\, d\mu', \quad 0 < \tau < \tau_0, \ -1 < \mu < 1, \tag{1}$$

$$S(\tau) = (1 - \varpi) \frac{\bar{n}^2 \bar{\sigma} T^4(\tau)}{\pi}, \tag{2}$$

with the boundary conditions

$$\begin{aligned} &I(0, \mu) = 0, \quad \mu > 0 \\ &I(\tau_0, -\mu) = 0, \quad \mu > 0 \end{aligned} \tag{3}$$

For the inverse problem, both the exit intensities, YI_m and ZI_m, and the exit temperatures, YT_m and ZT_m, at $\tau = 0$ and $\tau = \tau_0$, are measured, respectively. The temperature profile $T(\tau)$, or the source term $S(\tau)$, and the single scattering albedo profile, $\varpi(\tau)$, are to be estimated. The direct problem is solved with the Discrete Ordinate method [7].

It is noted that exit temperatures, T_m, are measured through using the two-color method [6]. In this method, $I_{\lambda_1, m}$ and $I_{\lambda_2, m}$, two values of monochromatic radiative intensities under two different wavelengths, λ_1 and λ_2, are measured. Then, the temperatures along the discrete directions can be calculated from the ratio of $I_{\lambda_1, m}$, and $I_{\lambda_2, m}$, neglecting the difference between the emissivities of the medium under the two wavelengths, as

* Corresponding author. Fax: +86-27-87545526.
E-mail address: hczhou@mail.hust.edu.cn (H.-C. Zhou).

PII: S0017-9310(00)00058-2

Nomenclature

I	radiation intensity
T	temperature
S	source term, Eq. (2)
τ	optical thickness
$\bar{\omega}$	single scattering albedo
$\bar{n}$	refractive index
$\bar{\sigma}$	Stefan–Boltzmann constant
μ	direction
m	ordinate direction
λ	wavelength
J	objective function
YI, ZI	measured exit radiation intensity
YT, ZT	measured temperature
∇J	gradient of the objective function
∇T	sensitivity coefficient vector for $J(\tilde{b})$ in Eq. (8)
δ^*	a small number
σ	standard deviation
ζ	random variable

$$T_m = C_2\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) \Big/ \left(\ln\frac{I_{\lambda_1, m}}{I_{\lambda_2, m}} - 5\ln\frac{\lambda_2}{\lambda_1}\right) \tag{4}$$

In order to obtain T_m, $I_{\lambda_1, m}$ and $I_{\lambda_2, m}$ can be calculated from Eq. (1) with two different monochromatic source terms listed as below

$$S_{\lambda_1}(\tau) = (1-\overline{\omega})\frac{C_1\lambda_1^{-5}}{\pi(e^{C_2/\lambda_1 T}-1)}$$
$$S_{\lambda_2}(\tau) = (1-\overline{\omega})\frac{C_1\lambda_2^{-5}}{\pi(e^{C_2/\lambda_2 T}-1)} \tag{5}$$

The boundary conditions take the similar forms like in Eq. (3).

The source term and scattering albedo are taken polynomial in the optical variable τ, respectively, i.e.

$$S(\tau) = \sum_{n_1=0}^{N_1} a_{n_1}\tau^{n_1} \qquad \overline{\omega}(\tau) = \sum_{n_2=0}^{N_2} b_{n_2}\tau^{n_2} \tag{6}$$

The estimation of $\tilde{a} = [a_0, a_1, \ldots, a_{N_1}]^{\mathrm{T}}$ and $\tilde{b} = [b_0, b_1, \ldots, b_{N_2}]^{\mathrm{T}}$ can lead to an estimation of $T(\tau)$ from Eq. (2).

For a given value of the scattering albedo $\bar{\omega}(\tau)$, the source term can be solved by minimizing the following objective function [5],

$$J(\tilde{a}) = \int_{-1}^{0}\left[I(0, \mu\,\tilde{a}) - YI(\mu)\right]^2 \mathrm{d}\mu + \int_{0}^{1}\left[I(\tau_0, \mu; \tilde{a}) - ZI(\mu)\right]^2 \mathrm{d}\mu \tag{7}$$

where $I(0, \mu; \tilde{a})$ and $I(\tau_0, \mu; \tilde{a})$ are estimated exit intensities at $\tau = 0$ and $\tau = \tau_0$, respectively, by using the estimated $\tilde{a}$. Thus, the estimation of $\tilde{a}$ is an optimization problem in (N_1 + 1)-dimensional space. The solving method adopted is the conjugate gradient method of minimization [5] and will not be described here in detail.

Here, $\tilde{b}$ is estimated using the following objective function

$$J(\tilde{b}) = \int_{-1}^{0}\left[T(0, \mu; \tilde{b}) - YT(\mu)\right]^2 \mathrm{d}\mu + \int_{0}^{1}\left[T(\tau_0, \mu; \tilde{b}) - ZT(\mu)\right]^2 \mathrm{d}\mu \tag{8}$$

In this function, $YT(\mu)$ and $ZT(\mu)$ are the measured exit temperatures at $\tau = 0$ and τ_0, respectively. $T(0, \mu; \tilde{b})$ and $T(\tau_0, \mu; \tilde{b})$ are the estimated exit temperatures at $\tau = 0$ and τ_0, respectively. Thus, the estimation of $\tilde{b}$ is also an optimization problem in (N_2 + 1)-dimensional space. In the iterative procedure,

$$\nabla T = \left(\frac{\partial T}{\partial b_0}, \frac{\partial T}{\partial b_1}, \ldots, \frac{\partial T}{\partial b_{N_2}}\right) \tag{9}$$

is the sensitivity coefficient vector. With a small increase, Δb_i, in b_i, the following approximate equation is used to determine ∇T

$$\frac{\partial T}{\partial b_i} \approx \frac{T(b_i + \Delta b_i) - T(b_i)}{\Delta b_i} \tag{10}$$

The following stopping criterion is adopted as below

$$J(\tilde{a}, \tilde{b}) = J(\tilde{a}) + kJ(\tilde{b}) \le \delta^* \tag{11}$$

where k (i.e., $k = 1/300^2$) is used to balance the magnitude difference between $J(\tilde{a})$ and $J(\tilde{b})$, and δ^* is a small number and chosen according to measurement errors.

The iterative procedure for the estimation method can be summarized as follows: Assume $\tilde{a}^k$ and $\tilde{b}^k$ are known at the k th iteration.

1. Using $\tilde{b}^k$, YI_m and ZI_m, estimate $\tilde{a}^{k+1}$.
2. Knowing $\tilde{a}^{k+1}$, YT_m and ZT_m, compute $\tilde{b}^{k+1}$.
3. Terminate the process if the specified stopping criterion, Eq. (11), is satisfied. Otherwise, return to step 1.

2. Results and discussion

Some numerical results were presented in order to examine the accuracy of the method for estimating the spatial distribution of the unknown source term, $S(\tau)$, or temperature, $T(\tau)$, and the unknown scattering albedo, $\varpi(\tau)$, from the data of the exit radiation intensities and the temperatures. A polynomial of degree six for the spatial distribution variation of $S(\tau)$ and a polynomial of degree four for the spatial distribution variation of $\varpi(\tau)$ were considered, that is

$$S(\tau) = 1 + 10\tau - 0.1\tau^2 + \tau^3 + \tau^4 - 10\tau^5 + 0.1\tau^6 \qquad (12)$$

$$\varpi(\tau) = 0.15 + 0.38\tau + 0.30\tau^2 - 1.00\tau^3 + 0.45\tau^4 \qquad (13)$$

The values of the optical thickness and the refractive index of the medium are chosen to be 1.0 and 1.0, respectively. For the solution of the direct problem, the discrete ordinates method with S-6 ordinate set was proved to be acceptable in precision [7]. The two wavelengths in the two-color method were chosen as 435.8 and 700 λm. In order to simulate the measured exit intensities, YI_m and ZI_m, and the temperatures, YT_m and ZT_m, containing measurement errors, random errors of standard deviation σ are added to the exact data. Thus, we have

$$D_{\text{measured}} = D_{\text{exact}} + \sigma\zeta \qquad (14)$$

where the range of random variables ζ is chosen as $-2.576 < \zeta < 2.576$, which represents the 99% confidence bound for the measured data.

In the estimation of the source profile and the scattering albedo, the initial values were chosen as $\tilde{a}^0 = \tilde{0}$ and $\tilde{b}^0 = 0.0\tilde{1}$. Three cases were examined in which one did not have measurement errors (i.e., $\sigma = 0.0$), and the other two had measurement errors of $\sigma = 0.03$ and $\sigma = 0.04$, respectively. With no measurement errors, no observable differences could be seen between the estimated and the exact data of the source term and the scattering albedo as in [5].

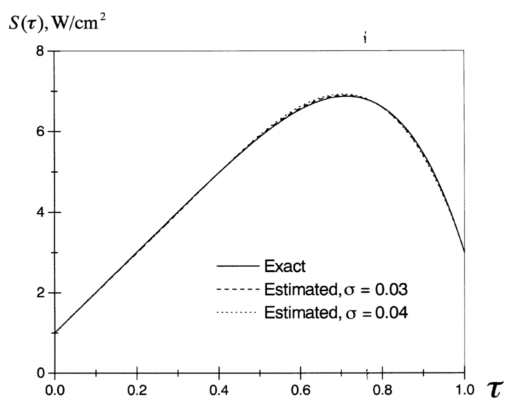

Fig. 1. Estimations of the source term $S(\tau) = 1 + 10\tau - 0.1\tau^2 + \tau^3 + \tau^4 - 10\tau^5 + 0.1\tau^6$ W/cm^2 with inverse analysis using simulated measurement data with $\sigma = 0.03$ and $\sigma = 0.04$.

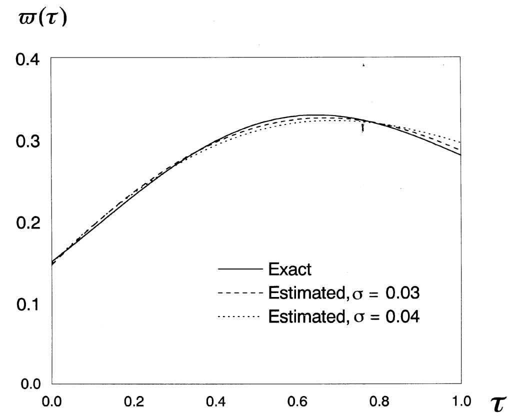

Fig. 2. Estimation of the scattering albedo $\overline{\omega}(\tau) = 0.15 + 0.38\tau + 0.30\tau^2 - 1.00\tau^3 + 0.45\tau^4$ with inverse analsis, using simulated measurement data with $\sigma = 0.03$ and $\sigma = 0.04$.

Fig. 1 shows the estimated $S(\tau)$ by inverse analysis with measurement errors of $\sigma = 0.03$ and $\sigma = 0.04$. Fig. 2 shows the estimated $\overline{\omega}(\tau)$ by inverse analysis with measurement errors of $\sigma = 0.03$ and $\sigma = 0.04$. It can be seen that in both the cases, the agreements between the exact and the estimated data are excellent. So the simultaneous estimation of the profiles of the temperature and the scattering albedo is acceptable.

The results indicate that exit temperatures measured with the two-color method contain more spectroscopic information about the radiative transfer process inside the gray medium. For non-gray media, this treatment may also be useful after careful extensions.

Acknowledgements

The present work was supported by the National Natural Science Foundation of China (No. 59606007), National Key Project for Fundamental Research, The Ministry of Science and Technology of China (G19990222) and the State Key Laboratory of Clean Coal Combustion of Tsinghua University, Beijing.

References

[1] C.E. Siewert, On the inverse problem for a three-term

phase function, J. Quant. Spectrosc. Radiant. Transfer 22 (1979) 441–446.

[2] S. Subbramaniam, M.P. Menguc, Solution of the inverse radiation problem for inhomengeous and anisotropically scattering media using a Monte Carlo technique, Int. J. Heat Mass Transfer 34 (7) (1991) 253–266.

[3] N.J. McCormick, Inverse radiative transfer problems: a review, Nuclear Science and Engineering 112 (1992) 185–198.

[4] R. Viskanta, Radiation heat transfer: interaction with conduction and convection and approximate methods in radiation, in: Proceedings of the Seventh International Heat Transfer Conference, Munich, Germany, 1982, pp. 103–121.

[5] H.Y. Li, M.N. Ozisik, Identification of the temperature profile in an absorbing, emitting, and isotropically scattering medium by inverse analysis, Journal of Heat Transfer 114 (1992) 1060–1063.

[6] B.C. Young, D.P. McCollor, B.J. Weber, M.L. Jones, Temperature measurements of beulah lignite char in a novel laminor flow reactor, Fuel 67 (1) (1988) 40–44.

[7] W.A. Fiveland, Discrete ordinate methods for radiative heat transfer in isotropically and anisotropically scattering media, Journal of Heat Transfer 109 (1987) 809–812.